An Energy-based Method for the Assessment of Battery and Battery-Ultracapacitor Energy Storage Systems in Pulse-load Applications

Yiou He, Richard Y. Zhang and John G. Kassakian
Department of Electrical Engineering & Computer Science
Massachusetts Institute of Technology
Cambridge, MA, 02139, USA
{yiouhe, ryz, jgk}@mit.edu

***Abstract*—A new energy-based method is described for studying the performance of battery or battery-ultracapacitor energy storage systems (ESS) in pulse load applications. The method estimates the total energy available in each ESS from a small number of discrete measurements at different states-of-charge. This considerably reduces experiment time, when compared to slowly depleting the ESS over the course of several hours. To illustrate the use of this method, experiments are performed for nickel-metal hydride (NiMH) batteries and lithium-iron phosphate (LiFePO4) batteries as well as their corresponding battery-ultracapacitor paralleled systems, under various pulse-load profiles and with different ratios of capacitor rating to battery energy. The results show accuracy of the method on the order of 0.5% and the improvements in total energy by incorporating ultracapacitors range from 3% to 47%, depending upon load profiles, experimental conditions and battery-ultracapacitor combinations.**

Keywords—Pulse load applications, battery, ultracapacitor, NiMH, LiFePO4, Li-ion, energy measurement.

I. INTRODUCTION

There are a variety of industrial and consumer applications that draw pulsed currents from the batteries. Examples include wireless communication circuits [1], cordless power tools [2], automotive engine starters [3], etc. Batteries are less efficient when operated in this way. When facing a sudden rise in power demand (and thus a high-current pulse), their relatively large internal resistance creates a large voltage drop so that they may fail to supply the required load voltage. To improve pulse load efficiency, ultracapacitors can be used in conjunction with the battery to form a *hybrid* energy storage system (ESS).

The performance of a battery or hybrid ESS under pulse load cannot be directly estimated from its datasheets, because nominal specifications are usually predicated on constant load conditions. Instead, experimental data is necessary to accurately determine performance. In the existing literature, experimental data are most commonly measured either over just a single current pulse ([4] – [7]), or over the entire charge cycle, by discharging the battery from full to depletion under the pulse load ([6] – [9]). The former method is useful for making quick estimates, but fails to provide an accurate picture of the long-term characteristics, since data is only taken at a single state-of-charge. The latter method greatly improves the accuracy and reliability of the data, but also dramatically increases experiment time. Far fewer data can be collected over any given span of time, and in cases of very light load or very low duty ratios, experiment time itself becomes prohibitive.

In this paper, a new method is proposed to balance the trade-off between the accuracy of the data and the time it takes to collect the data. In the end, our experiments show accuracy within ~0.5% of the "run-time" method, but requiring less than half of the experimental time.

The new method makes use of the *smoothed voltage*, denoted as Φ and defined to be a smooth function of charge, whose integral over charge yields the total output energy. Integrating a smooth function is a well-studied problem, with a classic solution and rigorous mathematical justification [10]. In particular, it is possible to compute the integral to high precision using just a few optimally-placed samples of the function. This way, the total output energy can be estimated from just a few *discrete measurements* of Φ at certain states-of-charge of the ESS. Increasing the number of discrete measurements can improve the accuracy of the integration at the expense of experiment time.

Our work is motivated by the study of ultracapacitors connected directly in parallel with batteries. By contrast, most existing research have focused on making the connection through a power electronics interface, in applications including electric vehicles ([11] - [13]) and renewable energy ([14], [15]). Unfortunately, these interfaces introduce an increase in complexity and cost to the ESS, and can also actually *reduce* efficiency during certain load profiles. This motivation is reflected in our choice of experiments throughout this paper. However, the measurement methodology as described is suitable for the study of all ESS under pulse load, including those with power electronics interfaces.

To experimentally validate this method, we measured Φ for ESSs constructed from Nickel-Metal Hydride (Ni-MH) cells, lithium-iron phosphate (LiFePO4) cells and different values of ultracapacitors. The pulse load profiles investigated range in amplitude from 4 A to 20 A, in pulse period from 50 ms to 200 ms and in duty cycle from 5% to 50%, as shown in Fig. 1.

Section II describes the definition and the derivation of the *smoothed voltage* Φ. Section III presents the experimental validation and the accuracy analysis of the method; Section IV uses the new method to experimentally assess the battery and the hybrid system under different pulse load profiles and with various battery-ultracapacitor combinations. The results also provide application data useful for choosing the right ultracapacitor to increase specific battery performance. Section V concludes the paper and addresses future work.

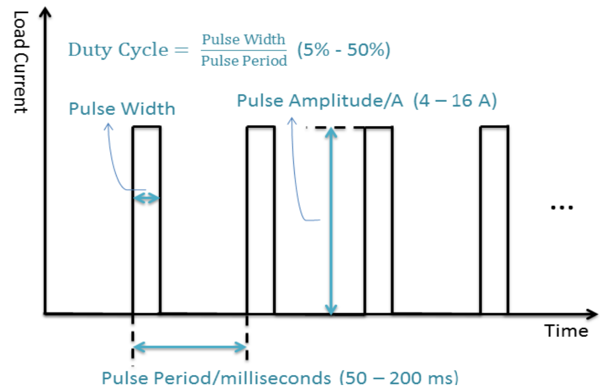


Fig. 1. Investigated pulse load profiles

II. THE SMOOTHED VOLTAGE METHOD

A. Formulation

The new method is based on two key assumptions:

- **Conservation of energy and charge.** There is no external energy or charge source other than the battery (or the hybrid) system. That is, as the battery is discharged, all charge is delivered to the load and any energy not delivered to the load is assumed to be dissipated as heat inside the source.
- **Weak history dependence.** Batteries show variability in their total available charge, internal resistance and discharge characteristics under different load profiles [16]. However, if a battery that has reached a particular state-of-charge under one load profile is then discharged under a lighter (heavier) load, then after a short "recover period", its voltage will rise (or fall) and a new total available charge (additional or reduced) at the second load profile can be obtained. This assumption is explained in further detail and experimentally validated in Section III.

From the first assumption, the energy transferred by a battery (or hybrid system) to a load can be written as an integral

$$E = \int v dq \tag{1}$$

where v is the battery terminal voltage and dq denotes an infinitesimal charge.

The equation explains why different load profiles may result in a differing amount of energy. With units of joules-per-coulomb, the terminal voltage v describes the potential energy available with every unit of charge drawn. Voltage is max at open circuit, and to maximize the amount of energy drawn, every bit of charge would ideally be delivered in a "trickled" manner at almost open-circuit. As the load current increases, the terminal voltage decreases, in a complicated fashion, and the amount of energy available per unit charge decreases.

In every case, we wish to measure the total energy delivered to the load. For this, (1) tells us that if we simply plot the voltage as a function of charge, *the area under the curve coincides with the amount of energy delivered to the load*. For this reason, we name the two-dimensional plane spanned by the variables v and q as the battery *phase plane*, and the curve traced by the voltage over this plane as the *voltage trajectory*.

Under a pulse-discharge load profile, the voltage trajectory contains numerous sharp spikes, making it highly non-smooth and difficult to integrate. To illustrate, an experimental trajectory for a LiFePO4 battery being discharged by a typical pulse load is shown in blue in Fig. 2. These spikes are formed at the end of every current-pulse, when the terminal voltage of the battery recovers to its (higher) open circuit voltage.

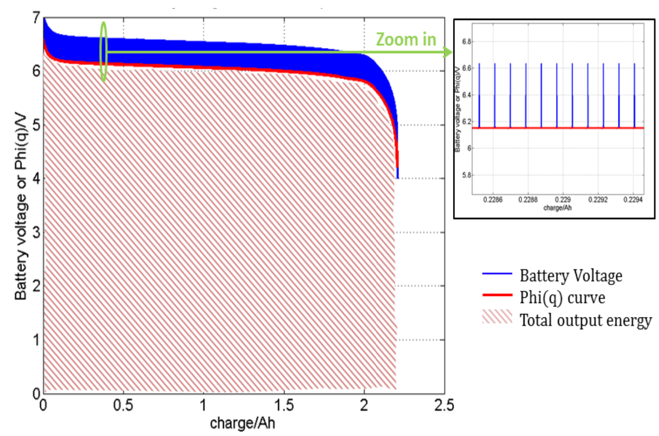


Fig. 2. Battery voltage (v), smoothed voltage (Φ) and the total output energy of a LiFePO4 battery being discharged with a typical pulse load profile

Our insight behind the smoothed voltage method is to realize that the area of each spike is negligibly small. We offer an intuitive explanation below. During the time when the terminal voltage recovers to its open circuit levels, the current drawn is nearly zero. And when current restarts, the voltage would rapidly stabilize to a relatively consistent value. The amount of charge actually drawn during this voltage transient is very small compared to the large amount of charge drawn during fixed-voltage discharge.

Hence, Φ is defined as the voltage trajectory with the spikes removed. Unlike the actual voltage, Φ is a smooth function of charge q, as shown in red in Fig. 2, and can be integrated much more easily. Yet the area underneath the smoothed voltage curve remains an excellent approximation to the total output energy, as in

$$E = \int v dq \approx \int \Phi(q) dq \tag{2}$$

since the areas of the spikes are negligible.

Finally, a note on measuring Φ. Rather than simply deleting spikes from the voltage waveform, accuracy is greatly improved by fitting $\Phi(q)$ to a measured change in energy. For each measurement, the battery is discharged from q_1 to q_2, and the energy discharged is computed by integrating the product of voltage and current. A single value of $\Phi = \Delta E/(q_2 - q_1)$ is then fitted over this entire interval.

B. Sampling and Integration

Once samples of Φ are obtained at charge values q_i, numerical integration can be performed by taking a "weighted sum" of the samples, as in

$$E_{out} \approx \int \Phi(q)dq \approx \sum w_i \Phi(q_i) \tag{3}$$

For example, the classic rectangle method takes samples at regularly-spaced q_i, and assign an equal weight to every sample. Unfortunately, in the case of the rectangle method, maintaining accuracy often requires numerous samples of the function being integrated.

We can do much better, by *optimizing* the sample locations q_i, noting that the smoothed voltage Φ is a smooth function of charge q. More specifically, it is a function that can be approximated to high accuracy by using a polynomial expansion, as in

$$\Phi(q) \approx c_0 + c_1 q + c_2 q^2 + c_3 q^3 \cdots \tag{4}$$

Integrating functions of this type is a classic problem with a well-known solution. It can be shown using rigorous mathematics that the best sample locations are at *the zeros of the Legendre polynomial*, and that the corresponding "weighted sum" is known as *Gauss-Legendre quadrature* [10]. Consequently, if samples are obtained at these optimal locations, then very few will be required to evaluate the integral to high precision. An extensive table of the Gauss-Legendre sample locations and weights can be found in [17].

Figure 3 illustrates an example of the optimal sample points of a $\Phi(q)$ curve. The $\Phi(q)$ curve is fitted by a 2n-1 order polynomial, which through the Gauss-Legendre quadrature theory then specifies that the required number of samples is n, e.g., a 15th order polynomial results in a sample size of 8. The weights and positions of the samples are produced through the theory and can be determined using a MATLAB function.

In this particular example, only 8 discrete measurements are needed to perform the integration to an accuracy of 0.05%. Therefore, the battery (or the hybrid) system under test can be discharged at a higher discharge rate between each successive pair of sample points (i.e. not necessarily in pulse discharge), and this will reduce the experiment time. Once Φ is measured at each sample point, the total output energy by the ESS is calculated using (3).

III. EXPERIMENTAL VALIDATION

A. Weak history dependence assumption

The gain of the new method in experimental time reduction lies in discharging the ESS at a higher rate between two sample points. One key assumption of this advantage is that after the high-rate discharge, if the ESS is pulse discharged for a long-enough period of time to let the transients settle down, it will behave the same (or with an ignorable error) as being pulse discharged from full charge to this sample point without the high-rate discharge in between. This assumption can also be interpreted as $\Phi(q)$ curve in the discharging steady state being largely independent of the charge/discharge history of the battery. Experiments were designed to validate the assumption.

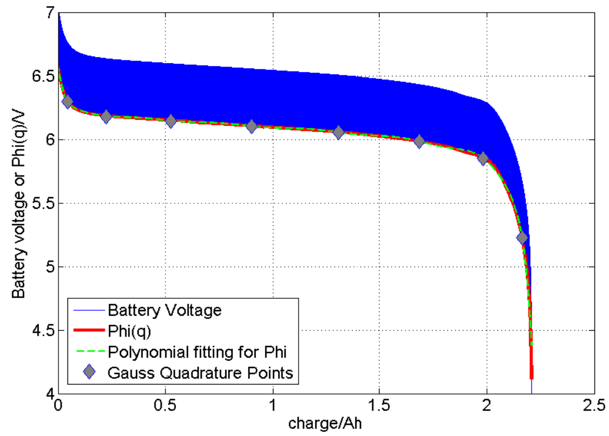


Fig. 3. Polynomial fit of the smoothed voltage (Φ) at the Gauss-Legendre quadrature nodes (a LiFePO4 battery being discharged at a typical pulse load profile)

1) Experimental setup

We built a charging/discharging/measurement apparatus to conduct all experiments. The connection of the batteries, the ultracapacitors and the charging/ discharging current sources is shown in Fig. 4. This apparatus can discharge the battery (or the hybrid) system at arbitrary pulse load profiles as well as measure and transfer voltage and current data to an external laptop simultaneously. The measurements and the controls were implemented with a 16-bit Analog-to-Digital converter AD7606 and a micro controller ATXmega192A3U.

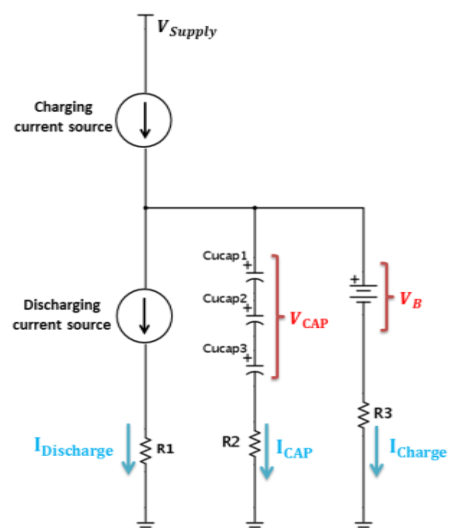


Fig. 4. Charging and discharging circuits

The battery-only system consists of 3.3 V LiFePO4 cells (two in series, 6.6 V in total) or 1.5 V Nickel-Metal Hydride (Ni-MH) cells (five in series, 7.5 V in total). In addition, three different values of ultracapacitors (1.667 F, 8.333 F and 16.67 F) are paralleled with the batteries to form the corresponding battery-ultracapacitor hybrid system. The specifications of the devices are shown in TABLE I.

TABLE I. SPECIFICATIONS OF THE DEVICES UNDER TEST

	Ni-MH [18]	LiFePO4 [19]	Ultracapacitors [20]
Norminal Voltage/cell	1.2 V	3.3 V	2.7 V
Capacity/cell	3.5 Ah	2.3 Ah	-
Capacitance/cell	-	-	5/25/50 F
Estimated ESR/cell	20 mΩ	10 mΩ	170/42/20 mΩ
Diameter/cell	18.2 mm	26 mm	10/16/18 mm
Length/cell	67 mm	66.5 mm	20/26/40 mm
Mass/cell	60 g	70 g	2.3/7.5/13 g
Manufacturer	Panasonic	A123	Maxwell
Part number	HHR370AH	ANR26650-M1	BCAP0005/0025/0050
Cells in Series	5	2	3

The smoothed voltage is denoted as Φ_B for the battery-only and Φ_H for the hybrid system. With reference to Fig. 4, Φ_B and $\Phi_H$at a certain q in the interval $[q_1, q_2]$, which corresponds to the time interval $[t_1, t_2]$, are calculated as

$$\Phi_B(q) = \left. \frac{\int_{t1}^{t2} V_B(t) i_{Charge}(t) dt}{\int_{t1}^{t2} i_{Charge}(t) dt} \right|_q \quad (5)$$

$$\Phi_H(q) = \left. \frac{\int_{t1}^{t2} (V_B(t) i_{Charge}(t) + V_{CAP}(t) i_{CAP}(t)) dt}{\int_{t1}^{t2} -i_{Discharge}(t) dt} \right|_q \quad (6)$$

2) Design of Experiments

The particular pulse load profile in the validation experiments was selected as a pulse amplitude of 8 A, a duty cycle of 20% and a period of 200 ms. The time increment $\Delta t = t_2 - t_1$ was chosen to be the length of ten pulses.

An approximation needs to be made in determining the total available charge, i.e., the charge duration of the polynomial fitting. In our experiments, because the pulse profiles range from 4 to 16 A and 5% to 50% duty cycle, the rms discharge rates are within 10 A. The available capacity of LiFePO4 batteries, is fairly constant as a function of discharge rate at these rates. Thus we consider the total available charge to be fixed at 2.2 Ah, as proved later in the experimental curves in Fig. 6. In the validation experiments, 8 sample points in the region of [0, 2.2 Ah] were calculated, as shown in Table II.

However for batteries the capacity of which varies dramatically with the discharge rates, such as in Ni-MH batteries, we need to adjust the total available charge at different load profiles by experience. For example, our Ni-MH batteries can only output 2.5 Ah total charge at the specified pulse load, but can output around 3.3 Ah when paralleled with 8.33 F ultracapacitors, as shown later in Fig. 9.

TABLE II. EIGHT SAMPLE POINTS AND WEIGHTS IN THE REGION OF [0, 2.2]

No.	GL nodes (Ah)	Weight
1	0.04368	0.11135
2	0.22366	0.24461
3	0.52191	0.34507
4	0.89822	0.39895
5	1.30177	0.39895
6	1.67808	0.34507
7	1.97633	0.24461
8	2.15631	0.11135

Two experiments were designed to run back-to-back on each system (the battery only and the hybrid system). The first one pulse discharged the system from 100% state-of-charge to its cut-off voltage; the second discharged the system by continuously discharging at 8 A between 8 periods of 750 pulses at the sample points. The phi value for a sample point was determined by applying (5) or (6) to the last 10 pulses at that point. The first 740 pulses are necessary to establish the "steady state."

For the LiFePO4 battery-only system, the $\Phi_B(q)$ curve of the first and the second experiment are shown in Figs. 2 and 5 respectively. The comparison of these two curves shows good consistency with our assumption, i.e., Φ at the sample nodes generated by the second experiment are coincident with the $\Phi_B(q)$ curve of the all-pulse discharge experiment, as shown in Fig. 6. The differences between the two $\Phi_B(q)$ curves at sample points are small enough so that the relative error of the integration is as low as 0.05%, as calculated in Table IV.

Figure 7 shows the comparison of the time needed by the two experiments. One can see that high-rate discharging between sample points reduces the experimental time by about 55% relative to pulse only discharging. Intuitively, when the number of sample points decreases, the experimental time will further decrease. This method is particularly useful when analyzing the ESS under pulse loads with small duty cycles. Experimental time can be largely reduced.

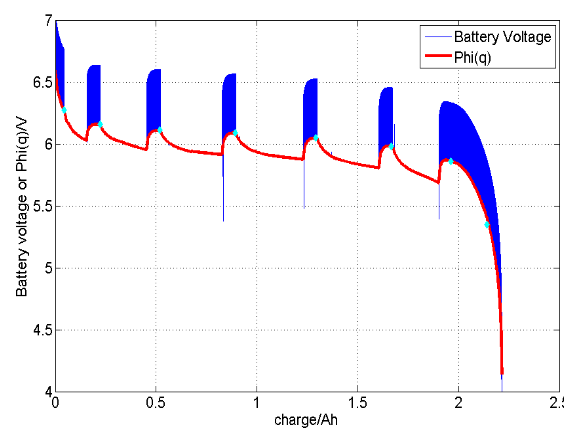


Fig. 5. Smoothed voltage curves ($\Phi_B(q)$ curves) and battery voltage curves for the LiFePO4 battery when pulse discharging at 8 A at 8 sample points and 8 A constant discharging in between

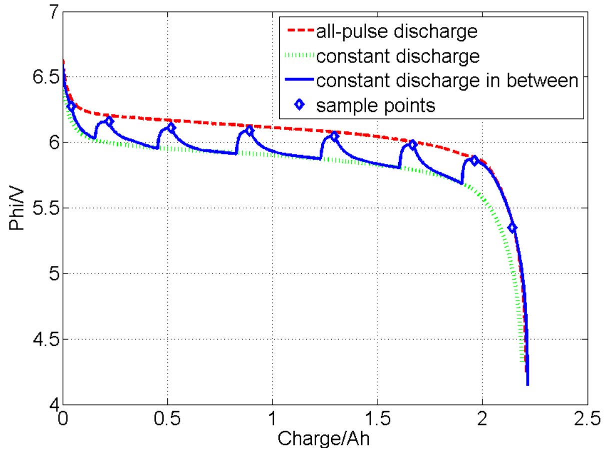


Fig. 6. Comparison of smoothed voltage curves for LiFePO4 battery-only system ($\Phi_B(q)$ curves)

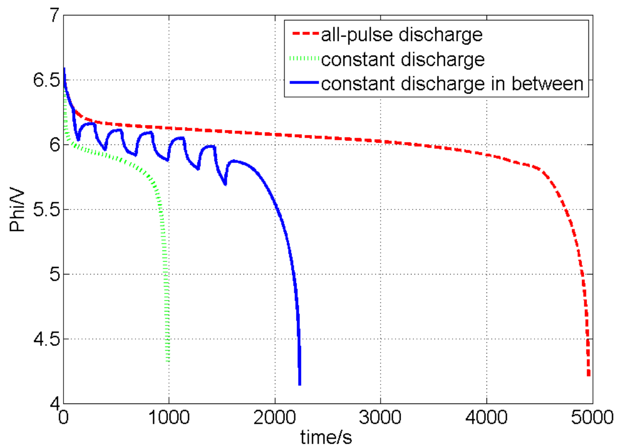


Fig. 7. Experimental time comparison of different discharge methods (LiFePO4 battery-only)

The hybrid system was also studied and the comparison of two $\Phi_H(q)$ curves for a LiFePO4 battery is shown in Fig. 8. The comparisons of $\Phi_B(q)$ and $\Phi_H(q)$ curves for Ni-MH batteries are shown in Fig. 9. Because the Ni-MH batteries are more sensitive to discharge conditions and history, these curves have a larger error than in the case of the LiFePO4 systems but still show a good consistency.

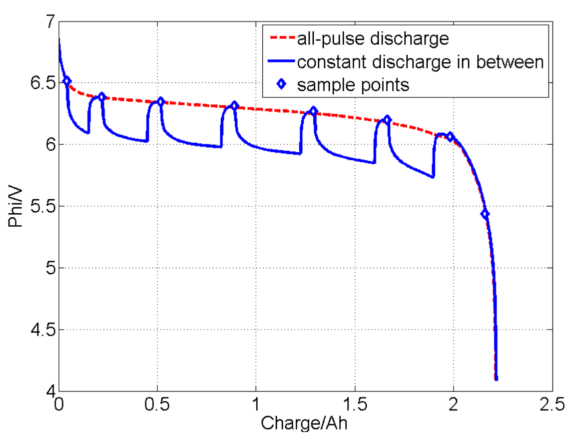


Fig. 8. Comparison of smoothed voltage curves for Li-ion-8.33 F hybrid system ($\Phi_H(q)$ curves)

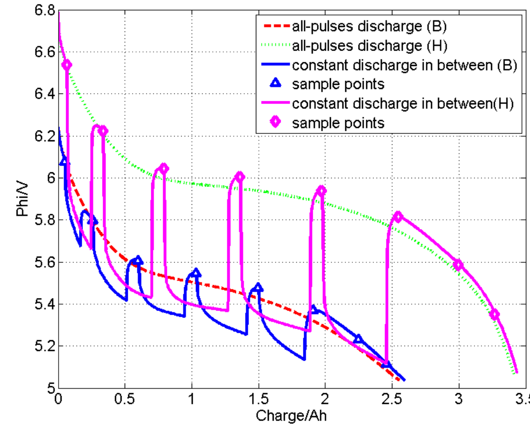


Fig. 9. Comparison of smoothed voltage curves ($\Phi_B(q)$ and $\Phi_H(q)$ curves for Ni-MH battery-only and hybrid system respectively)

B. Errors when using the smoothed voltage method

There are two categories of errors when using the smoothed voltage method to assess the total output energy: errors in measuring the positions of the sample points (charge errors or horizontal errors) and those in measuring Φ at each node (Φ errors or vertical errors).

Charge errors appear when the experimental sample points are not exactly equal to the designed sample points. It includes errors in real-time charge measurement (or state-of-charge measurement), errors in estimating the total available charge in the ESS, etc. Charge errors also contribute to Φ errors. Polynomial interpolation [10] was used to reduce this contribution to Φ errors, as follows. Note q_i as the ith designed sample points and $\Phi(q_i)$ is expected to be its corresponding Φ, however q_i' and $\Phi(q_i')$ are measured from the experiments. $\Phi(q_i)$ is calculated from q_i, q_i' and $\Phi(q_i')$, as shown in TABLE. III.

TABLE III. USE LEGENDRE INTERPOLATION TO ADJUST Φ VALUE (LI-ION BATTERY ONLY EXPERIMENT)

No.	q_i (Ah)	q_i' (Ah)	$\Phi(q_i')$ (Wh)	$\Phi(q_i)$ (Wh)
1	0.04368	0.04276	6.27145	6.27037
2	0.22366	0.22133	6.15652	6.15581
3	0.52191	0.51769	6.10938	6.10897
4	0.89822	0.89165	6.08842	6.08819
5	1.30177	1.2929	6.04701	6.04532
6	1.67808	1.66686	5.98122	5.97987
7	1.97633	1.96298	5.85731	5.83943
8	2.15631	2.14142	5.34956	5.26922

Other contributors to Φ errors include experimental hardware errors and transient errors. The former consists of the tolerance of the shunt resistors, the ADC resolution, the offset of amplifiers, etc. For each node, the experimental hardware error is approximately 0.6% for the battery-only system and 1.5% for the hybrid system [21]. The transient errors result from the "recovery" process from the high-rate discharge to the pulse discharge. Intuitively, longer recovery time yields smaller transient errors.

We also analyzed the sensitivity of the total output energy E_{out} to these errors. At each sample point q_i, assume there is a vertical relative error $\delta\Phi(q_i)$ and a horizontal relative error δq_i. δq_i is equivalent to δw_i, relative errors of the weighting factor. According to (3), the relative error in the total output energy contributed by each node can be derived,

$$\delta E_{out\_i} \approx \frac{w_i \Phi(q_i)}{E_{out}} (\delta\Phi(q_i) + \delta w_i) \tag{7}$$

$$\delta E_{out} \approx \sum \delta E_{out\_i}$$

For batteries with a relatively flat discharge characteristic, the sample points with larger weight, such as the 4th and 5th nodes in Table II, usually contribute more to the error.

C. Accuracy and time: the smoothed voltage method vs. traditional methods

We compared the accuracy of the new method and the traditional run-time method for both the battery and the hybrid systems. In each case, the total output energy E_{out} calculated by (3) was compared with E_{out_old}, the energy calculated by integrating the power over time in the all-pulse discharge experiment. Examples of LiFePO4 are shown in Table IV (LiFePO4 No. 1&2). For either the battery alone or the hybrid system, the new method is accurate to within 0.5%. Examples of Ni-MH are shown in the same table (NiMH No. 1-5) and they present a larger error.

In addition, the same experiments have been conducted to compare the accuracy and the experimental time when varying the number of sample points, as shown in Table IV (LiFePO4 No. 3&4) and Fig. 10. Increasing the number of discrete measurements will increase the accuracy of the total output energy but requires considerably longer experimental time.

TABLE IV. ACCURACY OF THE SMOOTHED VOLTAGE METHOD

Batt. No.		Pnts (#)	E_{out_old} (Wh)	E_{out} (Wh)	Abs. Err. (Wh)	Rel. Err.
LiFePO4 1&2	Batt. (Fig. 6)	8	13.238	13.232	0.0068	0.05%
	Hyb. (Fig. 8)		13.722	13.671	0.0513	0.374%
Ni-MH 1-5	Batt. (Fig. 9)		13.968	14.035	0.067	0.48%
	Hyb. (Fig. 9)		19.97	20.13	0.16	0.81%
LiFePO4 3&4	Batt.	8	12.738	12.778	0.04	0.313%
	Batt.	7	12.738	12.787	0.0493	0.39%
	Batt.	6	12.738	12.772	0.034	0.267%
	Batt.	5	12.738	12.784	0.0464	0.36%
	Batt.	4	12.738	12.78	0.042	0.33%
	Batt.	3	12.738	12.833	0.0953	0.75%
	Batt.	2	12.738	12.85	0.112	0.882%
	Batt.	1	12.738	12.832	0.094	0.745%

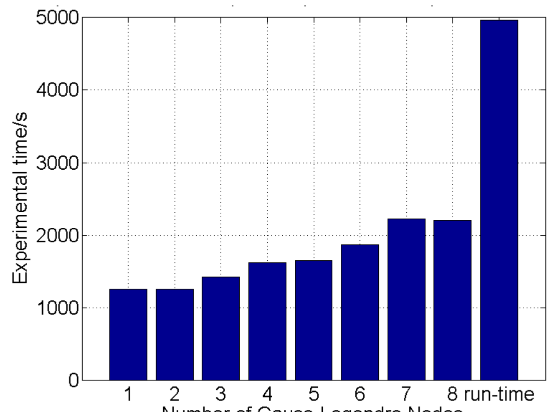


Fig. 10. Experimental time against numbers of discrete measurements taken (pulse discharge on LiFePO4 battery-only system)

In fact, when the number of discrete measurements equals one, the smoothed voltage is an equivalent “midpoint voltage”, the concept of which is always used in assessing the batteries' energy capacity at constant current discharge. Furthermore, the new method approaches the “run-time” method as the number of discrete measurements approaches the total pulse number. From Table IV and Fig.10, if the number of discrete measurements is chosen between 4 and 6, the accuracy can be kept within 0.5% but the experimental time is reduced to less than 40%. Thus the new method can be a good alternative to the traditional “run-time” method.

IV. BENEFITS OF BATTERY-ULTRACAPACITOR HYBRID SYSTEMS

By using the smoothed voltage method, investigations have been conducted on analyzing the performance of the battery and hybrid energy storage systems in high-current low-frequency pulse load applications. The investigations cover five degrees of freedom: pulse amplitude, pulse duty cycle, pulse period as well as battery type and capacitance value.

The benefit of a hybrid system over its battery-only system can be determined as how much more energy, relative to a battery only, the hybrid system can provide under combinations of these five degrees of freedom.

Each of the experiments consisted of fully charging the battery or hybrid system, then pulse discharging at 8 sample points and high-rate discharging between each successive pair of sample points. Accordingly, $\Phi(q)$ curves were generated and categorized into groups, as shown in Fig. 11 for a LiFePO4-based system. For this figure, the $\Phi(q)$ curves were generated using pulses whose features were kept the same except the pulse amplitude.

The total output energy in each case, i.e., the area enclosed by each of the $\Phi(q)$ curves and q axis, was calculated using (3) and compared. Figures 12 (a) to (d) show the total output energy of the LiFePO4-based system changing with pulse amplitude, duty cycle, period and capacitance respectively. Figures 13 shows the total output energy of the Ni-MH-based system changing with the capacitance.

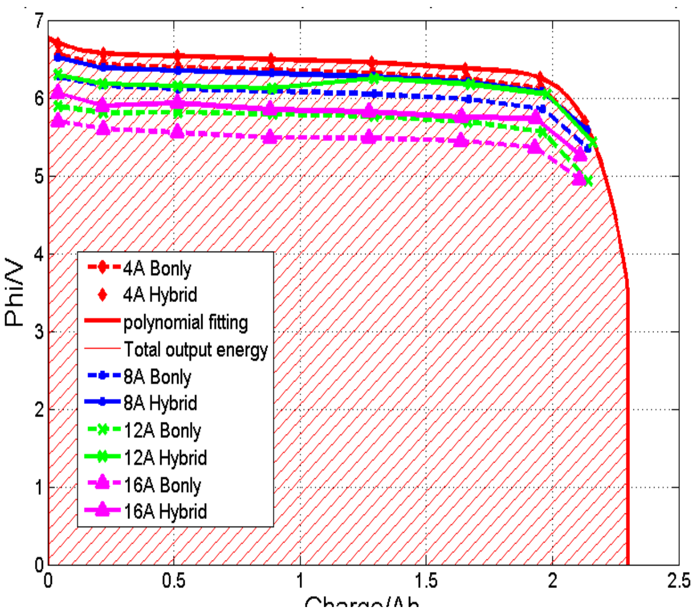
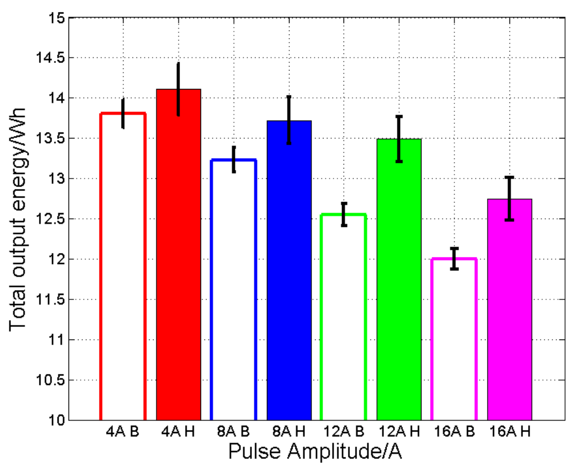
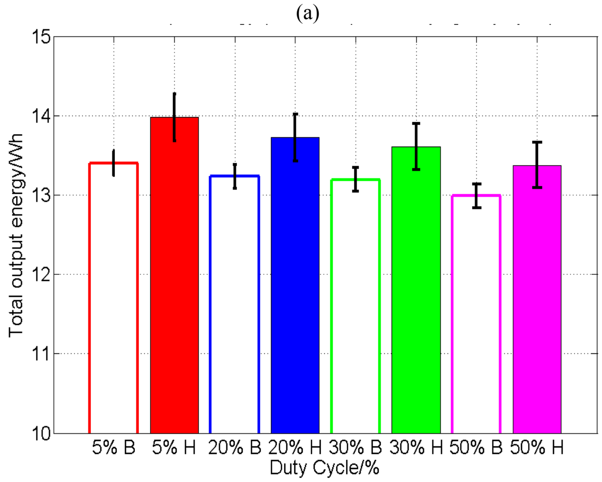


Fig. 11. Groups of smoothed voltgate curves (Φ(q)) when varying the pulse amplitude. The area under each Φ(q) curve is the corresponding total output energy. (LiFePO4 batteries)



(a)



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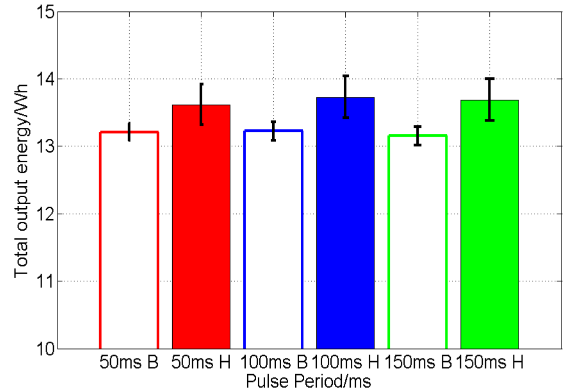
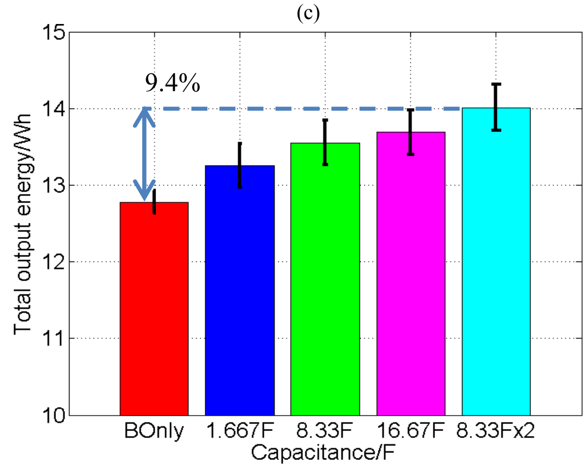


(c)



(d)

Fig. 12. Comparison of groups of the total output energy of LiFePO4 batteries/hybrids changing with different degrees of freedom {(a) - (d): the total output energy varying with the pulse amplitude, the duty cycle, the pulse period and the capacitance value;} (in all graphs, "B" means battery-only system, H means batteries and 8.33 F ultracapacitors paralleled system; in (d), 8.33Fx2 means two 8.33 F branches in parallel)

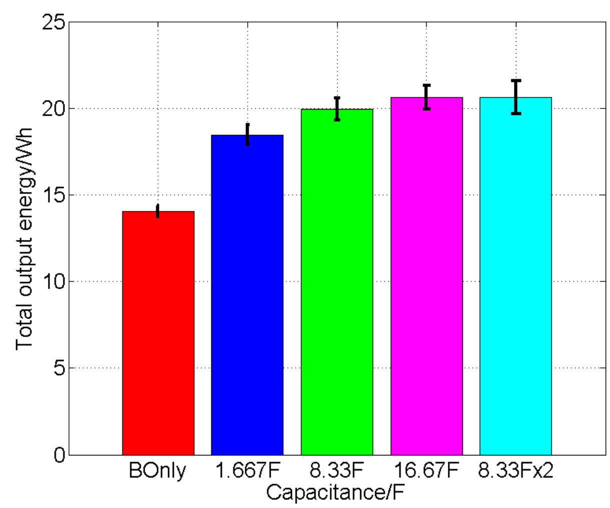


Fig. 13. Comparison of groups of the total output energy of Ni-MH batteries/hybrids changing with the capacitance value ("Bonly" means battery-only system, 8.33Fx2 means two 8.33 F branches in parallel)

From these results, we can conclude that:

Firstly, both the battery and the hybrid system will output more energy when the pulse amplitude is lower or the duty cycle is smaller. When the pulse period changes, however, the total output energy remains almost constant. This is mainly due to our pulse periods of interest being limited within 50 to 200 ms, but theoretically it should show an increasing trend when the pulse frequency further increases.

Secondly, in all cases the hybrid system is more beneficial than the battery-only system. And it presents more benefits in the cases where the load has higher pulse amplitude and smaller duty cycle.

Different combinations of the batteries and ultracapacitors yield differing improvements of the hybrid system over its battery-only counterpart. Generally when the batteries have larger internal resistance and the paralleled ultracapacitors have smaller internal resistance, the benefits are larger.

Eventually, the improved performance by incorporating ultracapacitors in the system is a consequence of their low internal resistance sharing current with the battery's internal resistance. It results in a lower battery rms current and a lower power dissipation in the battery, and thus a lower battery temperature and increased lifetime.

Among the cases we investigated, the improvement of $LiFePO_4$-based systems ranges from 2.7% to 9.4%. The best case is when two 8.33 F branches are connected in parallel and the system is discharged at a load with a pulse amplitude of 8A, a duty cycle of 20% and a period of 200ms. For Ni-MH battery, the improvement in the same case can reach to 47%.

V. CONCLUSION

A new methodology of accessing the total available energy in a battery (or a battery-ultracapacitor paralleled) energy storage system (ESS) in pulse load applications is proposed. The assumptions and the derivation of the method are presented. We also experimentally validate the method as well as analyze its error and accuracy. The method allows the total available energy in an ESS to be determined with high accuracy using a few discrete measurements and requires much less time than the traditional run-time method.

The method is used to experimentally measure the benefits of a battery-ultracapacitor paralleled system over its battery-only counterpart. Experimental results are presented for both $LiFePO_4$ and Ni-MH batteries and under various sets of load profiles. It is hoped that this method will be further used to assess the performance of other ESSs and under a wider set of experimental conditions.

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